STANDING WAVE ON A STRING

IDEA TO REMEMBER!

Every object resonates at a natural frequency!

OBJECTIVE:

Investigating the relationship between the frequency of the vibration and tension in the string to produce a standing wave.

MATERIALS:


Tape Measure


String and scissors


PASCO String Vibrator


PASCO 550 Interface


Banana jack cables


Pulley with rod


Right-angle clamp


Slotted weights


2x Table clamps


2x Rods


Digital scale


Weight hanger

CONCEPT:

Let's imagine we are designing a new guitar for your favorite musician. In order to produce the special sounds the musician wants, we have to manufacture the strings with a special weight, length, and tension so that the desired sound is created at the string's *natural frequency*, or **harmonic**, which is when the sound is loudest. To understand how to do this we should first study wave motion.

Wave motion is behind many phenomena—such as sound and light (see the *Real World Applications* section below). In nature, waves might be transverse, propagating perpendicular to the displacement (viz. a moving hill) or longitudinal, propagating parallel to the displacement (viz. a slinky). For our guitar, the strings produce a type of transverse wave called a **standing wave**.

Standing waves are generated when a consistent wave pattern propagates to a reflection point and then the reflected wave travels back in the opposite direction with the same *period*, *wavelength*, *frequency*. A phenomenon called **interference** allows these two, opposing waves to add together (constructive interference) and this can create a harmonic, where the *amplitude* doubles. Turn up the volume!

So, back to our guitar project, we notice the sound changes with the string length (that's why guitar strings have varying lengths!), with the string weight, and with the string tension. We then notice—at just the right combination—the sound gets louder, which we assume is the interference causing a harmonic! Therefore, we need to **understand how the harmonics correlate to the string density μ, frequency f, wavelength λ, wave speed v, and string tension F_T.**

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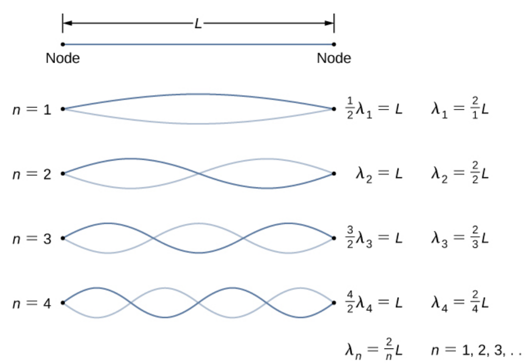
The linear density of the string μ is given as,

$$\mu = \frac{\text{mass of string}}{\text{sample length}} = \frac{m}{l} \tag{1}$$

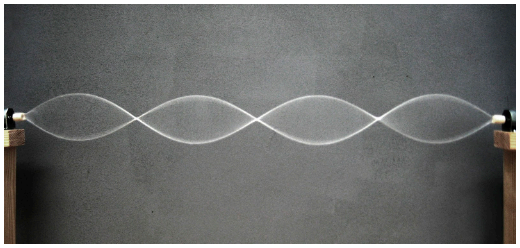
Where m is the mass of the string and l is the sample length. Experientially, we know that higher densities produce lower natural frequencies. Now, the interference that creates standing waves only occurs when there are two ends, which are a certain length L apart. The two ends will automatically become **nodes** (standing points). See Figure (1), where n is the harmonic number and λ is the wavelength—a full oscillation of the wave. Wavelength λ for n^{th} harmonic is shown as,

$$\lambda_n = \frac{2L}{n} \tag{2}$$

Notice, the sample length l for density is different than the end-to-end length L used to calculate wavelength λ. It is also clear that the number of nodes is equal to the harmonic number n + 1; $n = 1$ is the lowest natural frequency is called the *fundamental frequency* or **first harmonic**.



(a)



(b)

Figure 1: Illustrations of standing wave wavelengths.

THINK: Who is your favorite guitar player or guitar-playing band?

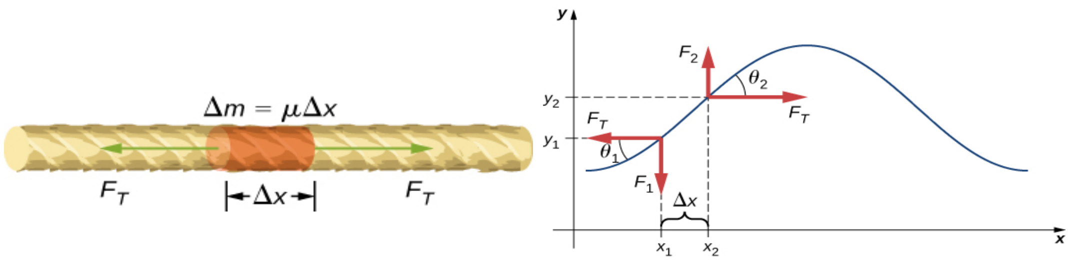
To find the wave speed we need to think logically and intuitively. The wave velocity v is the distance λ the wave travels in a given period T, and therefore, the frequency f of the sound produced is proportional to the wave speed v,

$$v = \frac{\lambda}{T} = \lambda f \tag{3}$$

Once again we should think logically and intuitively to find F_T. Selecting a small part of the string, draw a free-body diagram to analyze the forces. See Figure (2) for reference and continue reading. Two equal forces pull on the ends of our section due to the tension and restoring force in the string. From there we realize that our section becomes part of a curve when the string is plucked and a wave forms. Our two forces become two vectors at different positions along the string. The x-component forces are equal to the string tension F_T and cancel each other.

THINK: How do we find the y-components of the force in the string?

The y-components can be found by solving for the slope of the curve (the tangent line) *tan(θ)* for each force vector, creating a formula for slope at each point, Equation (4a) and (4b). The net force of the y-components then becomes (4c). Equating the net force to its Newtonian value (*ma*) allows us to substitute *m* for the



(a) (b)

Figure 2: A string under tension is plucked, causing a pulse to move along the string in the positive x-direction. [OpenStax]

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linear density μ, also substituting a for its derivative form, (4d). Next we can divide by the tension F_T and change in position Δx and derive an equation for the curve, (4e), where the Δx approaches zero. Solving that, we now have an equation for the slope of the wave, (4f), which looks very similar to the linear wave equation, (4g)—one of the most important equations in physics and engineering! Since the left of (4f) and (4g) is the same, we can now cancel the derivatives to find that the linear density of the guitar string μ and the tension in the string F_T determine the speed of the waves in the string, (5).

$$\frac{F_1}{F_T} = -\left(\frac{\partial y}{\partial x}\right)_{x_1} \quad (4a) \qquad \frac{F_2}{F_T} = \left(\frac{\partial y}{\partial x}\right)_{x_2} \quad (4b) \qquad F_{net} = F_1 + F_2 = F_T\left[\left(\frac{\partial y}{\partial x}\right)_{x_2} - \left(\frac{\partial y}{\partial x}\right)_{x_1}\right] \quad (4c)$$

$$F_T\left[\left(\frac{\partial y}{\partial x}\right)_{x_2} - \left(\frac{\partial y}{\partial x}\right)_{x_1}\right] = \mu\Delta x\frac{\partial^2 y}{\partial t^2} \quad (4d) \qquad \lim_{\Delta x \to 0}\frac{\left[\left(\frac{\partial y}{\partial x}\right)_{x_2} - \left(\frac{\partial y}{\partial x}\right)_{x_1}\right]}{\Delta x} = \frac{\mu}{F_T}\frac{\partial^2 y}{\partial t^2} \quad (4e)$$

$$\frac{\partial^2 y}{\partial x^2} = \boxed{\frac{\mu}{F_T}}\frac{\partial^2 y}{\partial t^2} \quad (4f) \qquad \frac{\partial^2 y(x,t)}{\partial x^2} = \boxed{\frac{1}{v^2}}\frac{\partial^2 y(x,t)}{\partial t^2} \quad (4g)$$

$$v = \sqrt{\frac{F_T}{\mu}} \quad (5)$$

We now have all of the equations and understanding to take the frequency specifications from our musician and create a special guitar for them!

Real World Applications

- People have understood this standing wave/harmonic phenomenon for thousands of years, building instruments with strings of different densities, tensions, and lengths to create unique sounds and ***music!***... The real question is: *who is the best guitar player?...*
- The most important example of a transverse wave are electromagnetic waves, or ***light!*** Similarly, the true power of electricity traveling through a wire is contained in the transverse and perpendicular electric and magnetic waves!

1) Using standing waves to ***levitate objects!***
2) How electricity actually flows—***transverse electromagnetic waves!***

PRECAUTIONS:

Be cautious while you hang the masses!

PROCEDURE:

1. ☐ Fill out the top information **and** complete the memory exercise—Question M1 and M2—on the worksheet.

2. ☐ REQUIRED: Read the *Concept* section.

3. ☐ Record the string's mass per unit length μ in kilograms per meter for Question 1 on the worksheet.

4. ☐ Assemble the setup as it is shown in Figure (3).

   4.1 With two rods clamps to opposing sides of the table, clamp the PASCO String Vibrator to a rod and tie a string through the hole in the metal tab.

   4.2 Attach the pulley to the table rod with a right-angle clamp so that the pulley rod is horizontal and the pulley wheel is vertical.

   4.3 Hang the other end your the string over the pulley and ensure that the string is level. If not, adjust the height of the PASCO String Vibrator or pulley.

   4.4 Tie a weight hanger to the end of the string under the pulley.

   4.5 Connect the PASCO String Vibrator with two banana wires through the 8V@400mA output channels of the PASCO 550 Interface.

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CONCEPT & PROCEDURE VIDEOS:

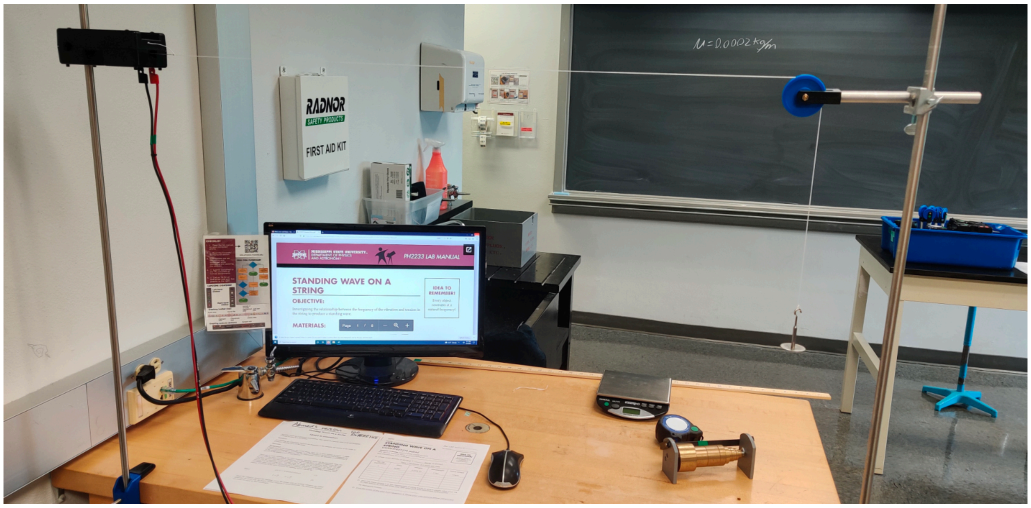

Figure 3: Setup of the experiment.

5. ☐ Measure the length *L* between the end points (from vibrator to pulley) and record for Question 2 on the worksheet.

6. ☐ Open the PASCO Capstone application, select *Hardware setup*, then click on the output channel to pick *output frequency sensor* (the yellow circle on the right).

7. ☐ Click *Signal Generator* and select a frequency (around 120Hz), and click "ON." (You can increase amplitude to 2–3 volts if amplitude is low.)

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8. ☐ Introduce some masses onto the weight hanger to adjust the tension until the fundamental frequency of the string matches with the Vibrator's fixed frequency.

9. ☐ Measure the weight *F* of the hanging mass.

10. ☐ Calculate wavelength λ and wave speed v (refer to the *Concept* section above).

11. ☐ Repeat steps 5–7 and fill Table 1 in your worksheet.

12. ☐ Answer Questions 3–6 on the worksheet.

13. ☐ Follow the **Let's THINK!** instructions below.

Let's THINK!

- **Ask questions**: What are you learning here?... Why is this Physics concept important and how can it be used?... What do you not understand?... (For more information on this Physics topic, scan the QR codes in the *Real World Applications* and at the start of the *Procedure* section.)
- **Discuss** the concept and demonstration with your partner to help each other understand better. Discussion makes learning active instead of passive!
- For **FULL PARTICIPATION [15 points]** you must call on the TA when you have finished your group discussion to answer some comprehensive questions. If you do not fully understand and the TA asks you to discuss more, you must call on them one more time to be dismissed with full marks.
- **CONCLUSION [10 points]**: In the Conclusion section at the end of the worksheet, write 3 or more sentences summarizing this concept, how this lab helped you understand the concept better, and the real world implications you see. Do you still have questions? If so, write those as well.

Updated Date	Personnel	Notes
2022.08	Chase Boone, Udeshika Perera, Ahmad Sohani, Brooks Olree	2022 Summer Improvement: Created new format.
2022.09, 2023.01	Chase Boone, Bryan Semon, Tawfik Gaballah	Improvements and clarifications.

Name: ______________________________ *PH2233* Section #: __________

Name: ______________________________ TA Name: ____________________

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WORKSHEET [70 points]

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Memory exercise [each 2 extra credit points]**:**

M1) Spring reaction force is proportional to __________________

M2) All oscillations have 3 things: __________________________

1) Linear mass density of string μ = ________ Kg/m [1 point]

2) Length L between the end points __________ m [1 point]

3) Mass of hanger ______ kg [1 points]

Table 1 [15 points, 0.5 point per cell]

No. Of Harmonics n	Mass M (kg)	Weight F (M*g, N)	Wavelength λ (m)	Wave speed v (m/s)

4) Plot the wave speed v vs the wavelength λ using Excel or graph paper. (Ask your TA to sign below to indicate that they saw your plot and approved.) [10 points]

TA Signature/Initials: ________________________

5) Find the slope of the plot from Question 3. What is the physical meaning of the slope? What are the units? [8 points]

6) Explain why the wavelength expression $\lambda_n = \frac{2L}{n}$ does not make sense for a non-integer n? (You can show it in mathematical expressions or pictures.) [6 points]

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7) Your favorite musician wants you to make a special guitar for them. They have provided the string lengths and notes that they want. Use the provided values to determine the string tension for proper tuning. [8 points, 2 points per cell]

Lengths, *L*	Notes	Note (fundamental) Frequencies, *f*	Linear mass densities, μ	String tension, *F*
1.0 m	D	293Hz	7.2g/m	
1.1 m	G	192Hz	6.2g/m	
1.1 m	B	246Hz	6.8g/m	
1.0 m	E	329Hz	7.5g/m	

8) Solve Equation (4f) in the *Concept* section to prove that the following equation is a possible solution to plot the harmonic (standing wave) curve. Show your work. [13 points]

$$y(x,t) = A\,sin(\omega t)\,sin(kx)$$

Conclusion

Write 3 or more sentences summarizing this concept, how this lab helped you understand the concept better, and the real world implications you see. Do you still have questions? If so, write those here as well. [10 points]

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